FreeFellow

FORMULA SHEET

FRM Part II

GARP · Financial Risk Manager

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FORMULAS

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TOPICS

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MARKET RISK

15 items

Sklar's theorem joint distribution decomposition

$F(x_1,\dots,x_n)=C(F_1(x_1),\dots,F_n(x_n))$ — F = joint CDF, F_i = marginal CDFs, C = copula on the unit hypercube

Put-call parity

$C-P=S-Ke^{-rT}$ — C = call price, P = put price, S = spot, K = strike, r = risk-free rate, T = time to expiry

Half-life of a Vasicek rate shock

$t_{1/2}=\ln 2/k$ — k = mean reversion speed; time for a rate shock to decay to 50% of its initial magnitude

Vasicek short-rate dynamics

$dr=k(\theta-r)\,dt+\sigma\,dW$ — k = mean reversion speed, θ = long-run rate, σ = volatility, dW = Brownian increment

Kolmogorov-Smirnov test statistic for PIT uniformity

$D=\sup_x|F_{emp}(x)-F_{unif}(x)|$ — F_emp = empirical CDF of PITs, F_unif = uniform [0,1] reference CDF

Standard error of empirical VaR quantile

$SE(VaR_c)\approx\frac{\sqrt{c(1-c)/n}}{f(VaR_c)}$ — c = confidence level, n = sample size, f(VaR_c) = density at the VaR quantile

Anderson-Darling test statistic for PIT goodness-of-fit

$A^2=-n-\sum_{i=1}^{n}\frac{2i-1}{n}[\ln u_{(i)}+\ln(1-u_{(n+1-i)})]$ — n = sample size, u_(i) = i-th order statistic of PITs

Vasicek conditional expected short rate

$E[r_T\mid r_t]=r_te^{-k(T-t)}+\theta(1-e^{-k(T-t)})$ — r_t = current rate, k = mean reversion speed, θ = long-run rate, T-t = horizon

Portfolio variance via VaR factor mapping

$\sigma_p^2=\mathbf{x}^T\mathbf{\Sigma}\mathbf{x}$ — x = vector of dollar exposures to each mapped risk factor, Σ = factor covariance matrix

Mean-reversion regression for equity correlation

$\Delta\rho_t=a(\bar\rho-\rho_{t-1})+\varepsilon_t$ — a = mean-reversion speed, ρ̄ = long-run correlation, ρ_{t-1} = lagged level, ε_t = shock

Regression-based hedge size with beta adjustment

$F_{hedge}=\beta\cdot\frac{DV01_{pos}}{DV01_{hedge}}\cdot F_{pos}$ — β = regression slope, DV01 = dollar value of a basis point, F = face amount

Lognormal value at risk

$\mathrm{VaR}_c=S_0\,[1-\exp(\mu-z_c\sigma)]$ — S_0 = position value, μ = mean log return, σ = log-return volatility, z_c = normal quantile at c

Bivariate Gaussian copula joint default probability

$P(\text{both default})=\Phi_2(\Phi^{-1}(p_1),\Phi^{-1}(p_2);\rho)$ — p_i = marginal default prob, ρ = asset correlation, Φ_2 = bivariate normal CDF

Correlation swap payoff

$\text{Payoff}=N\cdot(\rho_{\text{realized}}-K)$ — N = notional, ρ_realized = pairwise-average realized correlation over swap life, K = strike correlation

Parametric normal value at risk

$\mathrm{VaR}_c=-\mu+z_c\sigma$ — μ = mean return, σ = return standard deviation, z_c = one-sided standard normal quantile at confidence c (e.g., 2.326 at 99%)

CREDIT RISK

15 items

Single-month mortality from constant prepayment rate

$\mathrm{SMM} = 1 - (1 - \mathrm{CPR})^{1/12}$ — SMM = monthly prepayment rate, CPR = annualized constant prepayment rate

Tranche impairment fraction from pool loss

$\text{impairment} = \frac{\max(0, L_{pool} - A)}{D - A}$ — L_pool = pool loss, A = attachment, D = detachment; clip to [0, 1]

Mezzanine tranche price via base correlation

$V_{[A,D]} = V_{[0,D]} - V_{[0,A]}$ — V = tranche PV priced under each slice's own base correlation; A = attachment, D = detachment

Cumulative default probability with constant hazard rate

$P(\text{default by } t) = 1 - e^{-\lambda t}$ — λ = constant hazard rate, t = time horizon in years

Hazard rate implied by CDS spread

$\lambda \approx \frac{\text{CDS spread}}{1 - R}$ — λ = annualized hazard rate, R = recovery rate

Expected loss on a credit exposure

$\mathrm{EL} = \mathrm{PD} \times \mathrm{LGD} \times \mathrm{EAD}$ — PD = probability of default, LGD = loss given default, EAD = exposure at default

CDS-bond basis

$\text{basis} = s_{CDS} - s_{bond}$ — s_CDS = CDS spread, s_bond = bond credit spread over Treasury; negative basis = buy bond + buy protection

Bilateral CVA

$\mathrm{BCVA} = \mathrm{UCVA} - \mathrm{DVA}$ — UCVA = unilateral CVA (counterparty default charge), DVA = debt value adjustment (own-default benefit)

Overcollateralization (OC) test ratio

$\text{OC ratio} = \frac{\text{pool collateral}}{\text{tranche balance}}$ — failure diverts pool cash from mezzanine and equity to pay down senior

Damodaran country equity risk premium adjustment

$\text{ERP adj} = \text{Sovereign default spread} \times \frac{\sigma_{eq}}{\sigma_{bond}}$ — σ_eq = equity volatility, σ_bond = sovereign bond volatility

Single-loan unexpected loss with deterministic LGD

$\mathrm{UL} = \mathrm{EAD} \times \mathrm{LGD} \times \sqrt{\mathrm{PD}(1 - \mathrm{PD})}$ — PD = probability of default, LGD = loss given default, EAD = exposure at default

Single-factor model asset return

$R_i = \sqrt{\rho}\, M + \sqrt{1 - \rho}\, \varepsilon_i$ — M = common factor ~N(0,1), ε_i = idiosyncratic shock ~N(0,1) independent across firms, ρ = asset correlation

Default correlation between two obligors

$\rho_d = \frac{P(D_i \cap D_j) - PD_i \cdot PD_j}{\sqrt{PD_i(1 - PD_i)\, PD_j(1 - PD_j)}}$ — PD_i = marginal default probability, P(D_i ∩ D_j) = joint default probability

Debt service coverage ratio

$\mathrm{DSCR} = \frac{\text{Net operating income}}{\text{Debt service}}$ — net operating income = property cash flow, debt service = scheduled principal + interest payments

Unilateral CVA on a derivative trade

$\mathrm{CVA} \approx (1 - R) \sum_i \mathrm{EE}(t_i) \cdot \mathrm{PD}(t_{i-1}, t_i) \cdot D(t_i)$ — R = recovery, EE = expected exposure, PD = marginal default prob, D = discount factor

OPERATIONAL RISK AND RESILIENCE

10 items

Fault tree AND-gate probability for independent events

$P_{AND} = p_1 \times p_2$ — p_1, p_2 = probabilities of independent child events that must both occur for parent event

Fault tree OR-gate probability for two independent events

$P_{OR} = p_1 + p_2 - p_1 p_2 \approx p_1 + p_2$ — p_1, p_2 = probabilities of child events; approximation valid for small probabilities

RCSA multiplicative risk score

$\text{Score} = L \times I$ — L = likelihood rating (1-5 scale), I = impact rating (1-5 scale); applied to both inherent and residual risk

Basel total capital ratio

$\text{Total capital ratio} = \frac{T_1 + T_2}{\mathrm{RWA}} \geq 8\%$ — T1 = Tier 1 capital, T2 = Tier 2 capital, RWA = risk-weighted assets

Basel III SMA operational risk capital

$K_{SMA} = BI \times ILM$ — BI = Business Indicator (size proxy from financials), ILM = Internal Loss Multiplier (scales from 1 by 10-yr loss / BI)

Capital held by a bank under integrated risk management

$K_{held} = \max(K_{reg}, K_{econ})$ — K_reg = Basel regulatory capital floor, K_econ = internal economic capital at target confidence (typically 99.95-99.97%)

Adjusted RAROC with systematic-risk correction

$\text{Adjusted RAROC} = \text{RAROC} - \beta_E (R_M - R_F)$ — β_E = activity equity beta, R_M = market return, R_F = risk-free rate; compare to R_F (not cost of equity)

Risk-adjusted return on capital (RAROC)

$\mathrm{RAROC} = \frac{R - EL - E - T + K \cdot r_f}{EC}$ — R = revenues, EL = expected loss, E = expenses, T = taxes, K·r_f = capital charge income, EC = economic capital

Basel III output floor on RWA

$\mathrm{RWA}_{\mathrm{eff}} = \max(\mathrm{RWA}_{\mathrm{IRB}}, 0.725 \times \mathrm{RWA}_{\mathrm{SA}})$ — IRB = internal ratings-based RWA, SA = standardized-approach RWA, 0.725 = 72.5% floor

Basel 2.5 market risk capital charge

$MRC = \max(\overline{VaR} \cdot m_c, VaR_{\mathrm{prev}}) + \max(\overline{sVaR} \cdot m_s, sVaR_{\mathrm{prev}}) + IRC + CRM$ — m = supervisory multipliers, IRC = incremental risk charge, CRM = comprehensive risk measure

Liquidity-adjusted VaR

$\mathrm{LVaR} = \mathrm{VaR} + \frac{1}{2}P(\bar{s} + z\sigma_s)$ — P = position size, $\bar{s}$ = average proportional bid-ask spread, σ_s = spread volatility, z = stress quantile

Required liquid-asset buffer under stress

$B = \mathrm{NSO} \times k$ — NSO = net stressed outflow over survival horizon, k = management cushion factor (typically 1.10–1.25)

Duration gap with leverage adjustment

$D_{\text{gap}} = D_A - D_L \cdot \frac{L}{A}$ — D_A = asset duration, D_L = liability duration, L = total liabilities, A = total assets

Covered interest parity forward exchange rate

$F = S \cdot \frac{1+r_d}{1+r_f}$ — F = forward rate (domestic per foreign), S = spot rate, r_d = domestic interest rate, r_f = foreign interest rate

Change in equity from a duration-gap rate shock

$\Delta E \approx -D_{\text{gap}} \cdot \frac{\Delta r}{1+r} \cdot A$ — D_gap = duration gap, Δr = parallel rate shock, r = current rate, A = asset value

Contingent liquidity risk charge for committed lines

$\mathrm{Charge} = L \cdot d_{\text{stress}} \cdot c_{\text{HQLA}}$ — L = committed line size, d_stress = assumed stress drawdown rate, c_HQLA = HQLA opportunity cost

Historical average cost of funds

$\bar{r} = \frac{\sum_i B_i r_i}{\sum_i B_i}$ — B_i = balance of funding source i, r_i = rate paid on source i; blended carrying cost across the existing funding stack

Net stressed outflow over a defined horizon

$\mathrm{NSO} = \sum_i r_i D_i + d \cdot L - I$ — r_i = category run-off rate, D_i = deposit balance, d = line drawdown rate, L = undrawn commitments, I = reliable inflows

Repo cash advance after haircut

$\mathrm{Cash} = V_{coll} \times (1-h)$ — V_coll = market value of collateral pledged, h = haircut percentage; interest accrues on this cash amount, not on collateral value

Available funds gap

$\mathrm{Gap} = (\Delta L + D_{out} + S) - (\Delta D + L_{in})$ — ΔL = new loan demand, D_out = deposit run-off, S = contractual debt service, ΔD = new deposit growth, L_in = loan repayments

Repo repurchase price

$P_{repurchase} = P_{sale} \times \left(1 + r_{repo} \cdot \frac{n}{360}\right)$ — P_sale = post-haircut cash advance, r_repo = repo rate, n = days to maturity, 360 = money-market day-count

Basel III Liquidity Coverage Ratio

$\mathrm{LCR} = \frac{\mathrm{HQLA}}{\text{Net cash outflows over 30 days}} \geq 100\%$ — HQLA = high-quality liquid assets after haircuts and caps; denominator = stressed 30-day outflows minus capped inflows

Basel III Net Stable Funding Ratio

$\mathrm{NSFR} = \frac{\text{Available stable funding}}{\text{Required stable funding}} \geq 100\%$ — ASF = liabilities/capital weighted by tenor and stickiness; RSF = assets weighted by liquidity profile over 1-year horizon

RISK AND INVESTMENT MANAGEMENT

15 items

IRR approximation from TVPI and average cash-flow duration

$IRR \approx TVPI^{1/T} - 1$ — TVPI = total value to paid-in multiple, T = average duration of net cash flows in years

Unsmoothed true return from reported series

$R_{true,t} = \frac{R_{rep,t} - \rho R_{rep,t-1}}{1-\rho}$ — R_rep = reported return, ρ = first-order autocorrelation

Z-score of a single risk factor

$z_x = \frac{x-\mu_x}{\sigma_x}$ — x = observation, μ_x = historical mean, σ_x = historical standard deviation

Total value to paid-in multiple (TVPI)

$TVPI = \frac{D + NAV}{PIC}$ — D = cumulative distributions, NAV = remaining net asset value, PIC = paid-in (called) capital

Smoothed-return autoregression for illiquid assets

$R_{rep,t} = \alpha + \rho R_{rep,t-1} + \varepsilon_t$ — R_rep = reported return, ρ = first-order autocorrelation, ε = innovation

Mahalanobis distance for a multi-variable stress scenario

$D^2 = (x-\mu)^T \Sigma^{-1} (x-\mu)$ — x = observation vector, μ = mean vector, Σ = covariance matrix

Un-smoothed private credit volatility (IMF GFSR April 2024)

$\sigma_{true} \approx \sigma_{reported}/0.40$ — σ_reported = quarterly NAV-based vol, 0.40 = IMF-flagged smoothing-to-true ratio

Information ratio

$IR = \frac{\bar{R}_p - \bar{R}_b}{\sigma(R_p - R_b)}$ — R_p = portfolio return, R_b = benchmark return, σ(R_p − R_b) = tracking error

Grinold's fundamental law of active management

$IR \approx IC \cdot \sqrt{BR} \cdot TC$ — IC = information coefficient, BR = breadth (independent bets/year), TC = transfer coefficient

Distributions to paid-in multiple (DPI)

$DPI = \frac{D}{PIC}$ — D = cumulative cash distributions to LPs, PIC = paid-in (called) capital

Component VaR of a position

$CVaR_i = w_i \cdot MVaR_i$, with $\sum_i CVaR_i = VaR_{div}$ — w_i = dollar position size, MVaR_i = marginal VaR per dollar of position i

Multifactor model expected return

$E[R_i] = R_f + \sum_k \beta_{i,k} \lambda_k$ — R_f = risk-free rate, β_{i,k} = asset i's loading on factor k, λ_k = risk premium on factor k

Jensen's alpha (single-factor)

$\alpha = (R_p - R_f) - \beta(R_b - R_f)$ — R_p = portfolio return, R_b = benchmark return, R_f = risk-free rate, β = portfolio beta to benchmark

Diversified portfolio VaR

$VaR_{div} = z \cdot \sqrt{w^T \Sigma w}$ — z = confidence multiplier (1.645 at 95%), w = vector of dollar positions, Σ = covariance matrix of returns

Modigliani-squared (M²) risk-adjusted return

$M^2 = R_f + \frac{\sigma_m}{\sigma_p}(\bar{R}_p - R_f)$ — R_f = risk-free rate, σ_m = benchmark volatility, σ_p = portfolio volatility, \bar R_p = mean portfolio return

CURRENT ISSUES IN FINANCIAL MARKETS

3 items

BCBS Group 2 risk-weighted assets for unbacked crypto

$RWA = \text{Exposure} \times 1{,}250\% = \text{Exposure} \times 12.5$ — Exposure = bank position in unbacked crypto or non-qualifying stablecoin

Minimum CET1 capital required against Group 2 crypto exposure

$\text{Required CET1} = RWA \times 4.5\% = \text{Exposure} \times 12.5 \times 4.5\%$ — RWA = risk-weighted assets, 4.5% = Basel III CET1 minimum ratio

Effective dollar-for-dollar capital charge on Group 2 crypto

$\text{Required CET1} \approx \text{Exposure}$ — 1,250% weight is calibrated so capital held equals the full position size, effectively expensing it from regulatory capital