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FORMULA SHEET

FRM Part I

GARP · Financial Risk Manager

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FOUNDATIONS OF RISK MANAGEMENT

11 items

Annual CDS premium payment

$\text{Premium} = s \times N$ — s = CDS spread (decimal), N = notional protected. Example: 250 bps on \$800M = \$20M/year

Expected loss on a credit exposure

$EL = PD \times LGD \times EAD$ — PD = probability of default, LGD = loss given default, EAD = exposure at default

CAPM expected return

$E[R_i] = R_f + \beta_i(E[R_m] - R_f)$ — R_f = risk-free rate, β_i = asset beta, E[R_m] = expected market return

CDS spread approximation

$s \approx PD \times LGD$ — s = annualized CDS spread, PD = annual probability of default, LGD = loss given default (1 - recovery)

Beta of an asset

$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2}$ — Cov(R_i,R_m) = covariance of asset and market returns, σ_m² = market return variance

Jensen's alpha

$\alpha = \bar{R}_p - [R_f + \beta_p(\bar{R}_m - R_f)]$ — R̄_p = realized portfolio return, R_f = risk-free rate, β_p = portfolio beta, R̄_m = market return

Aggregate firm-wide economic capital variance

$\sigma^2_{firm} = \sum_i \sigma_i^2 + 2\sum_{i<j} \rho_{ij}\sigma_i\sigma_j$ — σ_i = standalone EC of silo i, ρ_ij = pairwise correlation between silos i and j

Factor model return decomposition

$R_i = E[R_i] + \beta_{i,1}f_1 + \cdots + \beta_{i,k}f_k + \varepsilon_i$ — f_k = zero-mean factor surprise, β_{i,k} = factor loading, ε_i = idiosyncratic noise

Risk-adjusted return on capital (RAROC)

$RAROC = \frac{\text{Risk-adjusted return}}{\text{Economic capital}}$ — numerator = return net of EL and funding costs, denominator = economic capital allocated

Fama-French three-factor model

$E[R_i] - R_f = \beta_{i,M}(R_M - R_f) + \beta_{i,SMB} \cdot SMB + \beta_{i,HML} \cdot HML$ — R_M = market return, SMB = small minus big, HML = high minus low book-to-market

APT multifactor expected return

$E[R_i] = R_f + \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2 + \cdots + \beta_{i,k}\lambda_k$ — R_f = risk-free rate, β_{i,k} = asset i loading on factor k, λ_k = factor k risk premium

AR(1) mean-reverting level

$\mu = \frac{\alpha}{1-\phi}$ — α = intercept, φ = AR(1) coefficient with |φ| < 1, μ = unconditional long-run mean

Continuously compounded (log) return

$r = \ln(P_t/P_{t-1}) = \ln(1+R)$ — P_t = price at time t, P_{t-1} = prior price, R = simple return over the period

Sample variance with Bessel's correction

$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ — X_i = observation i, $\bar{X}$ = sample mean, n = sample size

Bayes' rule

$P(A \mid B) = \frac{P(B \mid A)\,P(A)}{P(B)}$ — P(A) = prior, P(B|A) = likelihood, P(B) = marginal evidence

Expected value of a lognormal random variable

$E[X] = e^{\mu+\sigma^2/2}$ — μ = mean of ln(X), σ² = variance of ln(X); the σ²/2 is the convexity adjustment

F1 score for binary classification

$F_1 = \frac{2 \cdot P \cdot R}{P+R}$ — P = precision = TP/(TP+FP), R = recall = TP/(TP+FN), TP/FP/FN from confusion matrix

Law of total probability (two-event partition)

$P(B) = P(B \mid A)P(A) + P(B \mid A^c)P(A^c)$ — A and A^c partition the sample space, P(B|·) = conditional probability of B

Adjusted R-squared

$\bar{R}^2 = 1 - (1-R^2)\cdot\frac{n-1}{n-k-1}$ — R² = unadjusted R-squared, n = sample size, k = number of regressors

OLS slope estimator (single regressor)

$\hat{\beta}_1 = \frac{\mathrm{Cov}(X,Y)}{\mathrm{Var}(X)}$ — Cov(X,Y) = sample covariance of X and Y, Var(X) = sample variance of X

AR(1) h-step-ahead forecast

$E[Y_{t+h}] = \mu + \phi^h(Y_t - \mu)$ — μ = mean-reverting level, φ = AR coefficient, h = forecast horizon, Y_t = current value

Pearson correlation coefficient

$\rho_{X,Y} = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \sigma_Y}$ — Cov = covariance of X and Y; σ_X, σ_Y = standard deviations; ρ ∈ [−1, 1]

Square-root-of-time volatility scaling

$\sigma_T = \sigma_1\sqrt{T}$ — σ_T = T-period volatility, σ_1 = one-period volatility, T = number of periods (assumes iid returns)

Variance of a linear transformation

$\mathrm{Var}(aX+b) = a^2\,\mathrm{Var}(X)$ — X = random variable, a = scale factor, b = shift constant

Omitted variable bias in a short regression coefficient

$\mathrm{bias}(\hat{\beta}_1) = \beta_2 \cdot \delta$ — β_2 = true coefficient on omitted X_2, δ = slope from regressing X_2 on included X_1

Variance of a two-asset weighted sum

$\mathrm{Var}(aX+bY) = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\,\mathrm{Cov}(X,Y)$ — a, b = weights; σ = standard deviation; Cov = covariance

Variance inflation factor for regressor j

$\mathrm{VIF}_j = \frac{1}{1-R_j^2}$ — R²_j = R-squared from regressing X_j on the other regressors; VIF > 10 commonly flags collinearity

One-sample t-statistic for testing a mean

$t = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$ — $\bar{X}$ = sample mean, μ_0 = hypothesized mean, s = sample std dev, n = sample size

Box-Pierce Q-statistic for residual autocorrelation

$Q_{BP} = T\sum_{k=1}^{m}\hat{\rho}_k^2$ — T = sample size, m = number of lags tested, ρ̂_k = sample autocorrelation of residuals at lag k

LASSO regression penalized objective

$\min_\beta \sum_i (y_i - \hat{y}_i)^2 + \lambda\sum_j |\beta_j|$ — y_i = observed, ŷ_i = predicted, β_j = coefficient j, λ = L1 penalty strength

Ridge regression penalized objective

$\min_\beta \sum_i (y_i - \hat{y}_i)^2 + \lambda\sum_j \beta_j^2$ — y_i = observed, ŷ_i = predicted, β_j = coefficient j, λ = L2 penalty strength

Two-sample t-statistic for difference of means

$t = \frac{\bar{X}_1-\bar{X}_2}{\sqrt{s_1^2/n_1+s_2^2/n_2}}$ — $\bar{X}_i$ = sample mean i, s_i = sample std dev i, n_i = sample size i

Jarque-Bera test statistic

$JB = \frac{n}{6}\left[S^2 + \frac{(K-3)^2}{4}\right]$ — n = sample size, S = sample skewness, K = sample kurtosis; chi-square with 2 dof under normality

F-statistic for joint hypothesis test

$F = \frac{(\mathrm{SSR}_R - \mathrm{SSR}_U)/q}{\mathrm{SSR}_U/(n-k-1)}$ — SSR_R = restricted SSR, SSR_U = unrestricted SSR, q = restrictions, n = sample, k = regressors

European call lower bound (no dividend)

$c \geq \max(S_0 - Ke^{-rT},\ 0)$ — c = call price, S_0 = spot, K = strike, r = risk-free rate, T = time to maturity

European put lower bound (no dividend)

$p \geq \max(Ke^{-rT} - S_0,\ 0)$ — p = put price, K = strike, r = risk-free rate, T = time to maturity, S_0 = spot

Continuous-to-discrete compounding conversion

$R_c = m \ln(1 + R_m/m)$ — R_c = continuous rate, R_m = rate compounded m times per year, m = compounding frequency

Bull call spread maximum profit

$\text{Max profit} = K_2 - K_1 - D$ — K_1 = long (lower) call strike, K_2 = short (higher) call strike, D = net debit paid

Optimal number of futures contracts (untailed)

$N^* = h^* \cdot Q_A/Q_F$ — h* = optimal hedge ratio, Q_A = size of cash exposure, Q_F = futures contract size

Cost-of-carry forward price for a commodity

$F_0 = S_0 e^{(r+u-y)T}$ — S_0 = spot, r = risk-free rate, u = storage cost rate, y = convenience yield, T = time to delivery

Forward price on a non-income asset

$F_0 = S_0 e^{rT}$ — S_0 = spot price, r = continuously compounded risk-free rate, T = time to delivery in years

Relative purchasing power parity

$(S_1 - S_0)/S_0 \approx i_d - i_f$ — S_0 = current spot, S_1 = future spot, i_d = domestic inflation, i_f = foreign inflation

Put-call parity for European options (no dividend)

$c + Ke^{-rT} = p + S_0$ — c = call price, p = put price, K = strike, r = risk-free rate, T = time to maturity, S_0 = spot price

Long straddle breakeven prices

$S^* = K \pm (c + p)$ — K = common strike, c = call premium, p = put premium; profit if |S(T) - K| > c + p

Bond price change with duration and convexity

$\Delta P/P \approx -D_{mod}\Delta y + \frac{1}{2}C(\Delta y)^2$ — D_mod = modified duration, C = convexity, Δy = yield change

Uncovered interest rate parity expected future spot

$E[S_T] = S \cdot (1 + r_d)/(1 + r_f)$ — E[S_T] = expected future spot, S = current spot, r_d = domestic rate, r_f = foreign rate

Conditional prepayment rate from single monthly mortality

$CPR = 1 - (1 - SMM)^{12}$ — SMM = single monthly mortality, CPR = annualized conditional prepayment rate

Forward price with continuous income yield

$F_0 = S_0 e^{(r-q)T}$ — S_0 = spot, r = risk-free rate, q = continuous income yield (dividend or foreign rate), T = time to delivery

Modified duration from Macaulay duration

$D_{mod} = D_{Mac}/(1 + y/m)$ — D_Mac = Macaulay duration, y = yield, m = compounding periods per year

Cheapest-to-deliver delivery cost

$\text{Cost} = \text{Quoted price} - (F \times CF)$ — Quoted price = clean bond price, F = futures settlement, CF = conversion factor

Tailed number of futures contracts

$N^*_{\text{tailed}} = h^* \cdot V_A/V_F$ — V_A = Q_A × S (dollar value of exposure), V_F = Q_F × F (dollar value of one futures contract)

Number of index futures to adjust portfolio beta

$N^* = (\beta^* - \beta_0) \cdot P/F$ — β* = target beta, β_0 = current portfolio beta, P = portfolio value, F = index futures contract value

Covered interest rate parity forward rate

$F = S \cdot (1 + r_d)/(1 + r_f)$ — F = forward rate, S = spot, r_d = domestic interest rate, r_f = foreign interest rate (matched to horizon)

Duration-based futures hedge ratio

$N^* = \frac{P \cdot D_P}{V_F \cdot D_F}$ — P = portfolio value, D_P = portfolio duration, V_F = futures contract value, D_F = CTD bond duration

Swap value to fixed-receiver (two-bond method)

$V = B_{\text{fix}} - B_{\text{fl}}$ — B_fix = PV of fixed-rate bond (coupons + notional), B_fl = PV of floating-rate bond (next coupon + notional)

Fisher relation linking nominal and real rates

$(1 + r) = (1 + \rho)(1 + i)$ — r = nominal rate, ρ = real rate, i = expected inflation

Long forward payoff at maturity

$\text{Payoff} = S_T - K$ — S_T = spot price at maturity, K = forward strike price

Variance swap payoff at maturity

$\text{Payoff} = N_{var} \times (\sigma^2_{realized} - K^2_{var})$ — N_var = variance notional, σ²_realized = realized variance, K_var = strike volatility

Long put option payoff at maturity

$\text{Payoff} = \max(K - S_T, 0)$ — K = strike price, S_T = spot at maturity

Forward rate between two dates (continuous compounding)

$f(t_1, t_2) = \frac{s_2 t_2 - s_1 t_1}{t_2 - t_1}$ — s1, s2 = spot rates to t1 and t2, f = forward rate from t1 to t2

Black-Scholes-Merton call price (no dividends)

$C = S_0 N(d_1) - K e^{-rT} N(d_2)$ — S_0 = spot, K = strike, r = risk-free rate, T = maturity, N = standard normal CDF

Cox-Ross-Rubinstein up and down factors

$u = e^{\sigma\sqrt{\Delta t}},\ d = 1/u$ — σ = annualized volatility of the underlying, Δt = step length in years

BSM gamma for a European option (no dividends)

$\Gamma = \phi(d_1)/(S_0 \sigma \sqrt{T})$ — φ = standard normal density, S_0 = spot, σ = volatility, T = maturity

d_1 in the Black-Scholes-Merton formula (no dividends)

$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$ — S_0 = spot, K = strike, r = rate, σ = volatility, T = maturity

Unexpected loss on a single credit exposure (binomial default)

$UL = EAD \times LGD \times \sqrt{PD(1 - PD)}$ — EAD = exposure at default, LGD = loss given default, PD = probability of default

Risk-neutral probability in a binomial tree

$p = \frac{e^{r\Delta t} - d}{u - d}$ — r = risk-free rate, Δt = step length, u = up-factor, d = down-factor

Unconditional default probability under constant hazard rate

$P(\tau \le t) = 1 - e^{-\lambda t}$ — λ = constant hazard rate (annual default intensity), t = horizon in years, τ = default time

KR01 from key-rate duration

$\mathrm{KR01}_k = \mathrm{KRD}_k \cdot P \cdot 10^{-4}$ — KRD_k = key-rate duration at rate k, P = bond price; dollar change per 1 bp shift

Survival probability under constant hazard rate

$P(\tau > t) = e^{-\lambda t}$ — λ = constant hazard rate, t = horizon in years, τ = default time

Bond price from discount factors

$P = \sum_{i=1}^{n} c \cdot d(t_i) + P \cdot d(t_n)$ — c = coupon, P = principal, d(t) = discount factor for time t, n = number of payments

Basel total market-risk capital with stressed VaR

$K = \max(VaR, \overline{VaR}_{60d}) + \max(sVaR, \overline{sVaR}_{60d})$ — VaR = current VaR, sVaR = stressed VaR, 60d-bar = 60-day average

Expected Shortfall under normal returns

$\mathrm{ES}_c = \mu + \sigma \cdot \frac{\phi(z_c)}{1-c}$ — μ = mean, σ = std dev, φ(z_c) = standard-normal density at z_c, c = confidence level

Effective duration from shocked prices

$D_{\text{eff}} = \frac{P_- - P_+}{2 P_0 \Delta y}$ — P_- = price after yield drop, P_+ = price after yield rise, P_0 = base price, Δy = shock size

Parametric (delta-normal) VaR

$\mathrm{VaR}_c = -\mu + z_c \sigma$ — μ = expected return, σ = return standard deviation, z_c = one-sided standard-normal quantile at confidence c

Loss given default from recovery rate

$\mathrm{LGD} = 1 - \mathrm{RR}$ — LGD = loss given default (fraction of exposure lost), RR = recovery rate (fraction of par recovered after default)

Macaulay duration

$D_{\text{mac}} = \sum_{i=1}^{n} t_i \cdot \frac{PV(C_i)}{P}$ — t_i = time to cash flow i, PV(C_i) = present value of cash flow i, P = bond price

Effective annual rate from nominal rate

$\mathrm{EAR} = (1 + r/n)^n - 1$ — r = nominal annual rate, n = compounding periods per year

Dollar value of an 01 (DV01)

$\mathrm{DV01} \approx D_{\text{mod}} \times P \times 10^{-4}$ — D_mod = modified duration, P = bond price; dollar price change for a 1 bp yield change

GARCH(1,1) variance recursion

$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$ — ω = constant, α = ARCH weight on lagged squared return, β = GARCH weight on lagged variance

Conditional one-year default probability given survival

$P(\text{default in next year} \mid \text{survived}) = 1 - e^{-\lambda}$ — λ = constant annual hazard rate; value is the same each year under constant intensity

Vasicek single-factor credit VaR (Basel IRB)

$VaR_c = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(c)}{\sqrt{1-\rho}}\right)$ — PD = default probability, ρ = asset correlation, c = confidence level, Φ = standard-normal CDF