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FORMULA SHEET

Exam FM

SOA / CAS · Financial Mathematics

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FORMULAS

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TOPICS

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TIME VALUE OF MONEY

6 items

Accumulation factor — compound interest

$a(t) = (1+i)^t$

i = annual effective interest rate, t = time in years

Present value factor

$v = \dfrac{1}{1+i} = 1 - d$

d = annual effective discount rate

Relationship: \(d\), \(i\), \(v\)

$d = \dfrac{i}{1+i} = 1 - v, \quad v = \dfrac{1}{1+i}$

Nominal rate \(i^{(m)}\) vs effective rate

$\left(1 + \dfrac{i^{(m)}}{m}\right)^m = 1 + i$

m = compounding periods per year

Force of interest

$\delta = \ln(1+i)$

$e^\delta = 1 + i, \quad a(t) = e^{\delta t}$

Simple interest accumulation

$a(t) = 1 + it$

Used for short periods (< 1 year) and treasury bills

ANNUITIES AND NON-CONTINGENT CASH FLOWS

11 items

Annuity-immediate (end of period) PV

$a_{\overline{n}|} = \dfrac{1 - v^n}{i}$

n payments of 1 at end of each period; $v = (1+i)^{-1}$

Annuity-due (beginning of period) PV

$\ddot{a}_{\overline{n}|} = \dfrac{1 - v^n}{d} = (1+i)\, a_{\overline{n}|}$

Annuity-immediate — accumulated value

$s_{\overline{n}|} = \dfrac{(1+i)^n - 1}{i} = (1+i)^n\, a_{\overline{n}|}$

Annuity-due — accumulated value

$\ddot{s}_{\overline{n}|} = \dfrac{(1+i)^n - 1}{d}$

Perpetuity-immediate PV

$a_{\overline{\infty}|} = \dfrac{1}{i}$

Perpetuity-due PV

$\ddot{a}_{\overline{\infty}|} = \dfrac{1}{d}$

Deferred annuity PV

${}_{k|}a_{\overline{n}|} = v^k\, a_{\overline{n}|}$

(annuity starting k periods from now)

Increasing annuity-immediate PV

$(Ia)_{\overline{n}|} = \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}$

Payments: $1, 2, \ldots, n$

Decreasing annuity-immediate PV

$(Da)_{\overline{n}|} = \dfrac{n - a_{\overline{n}|}}{i}$

Payments: $n, n-1, \ldots, 1$

Continuously paid annuity PV

$\bar{a}_{\overline{n}|} = \dfrac{1 - e^{-\delta n}}{\delta}$

Mthly annuity-immediate PV

$a^{(m)}_{\overline{n}|} = \dfrac{1 - v^n}{i^{(m)}}$

mn payments of $1/m$ per period

LOANS

4 items

Prospective loan balance

$B_t = P\, a_{\overline{n-t}|}$

PV of remaining $n - t$ payments; P=level payment

Retrospective loan balance

$B_t = L(1+i)^t - P\, s_{\overline{t}|}$

L=original loan, P=level payment

Interest and principal portions of payment \(k\)

Interest: $I_k = i\, B_{k-1}$

Principal: $PR_k = P - I_k$

Sinking fund method — annual cost

Annual cost $= iL + \dfrac{L}{s_{\overline{n}|j}}$

i=loan rate, j=sinking fund rate, L=loan

BONDS

3 items

Bond price formula

$P = Fr\, a_{\overline{n}|} + Cv^n$

F=face, r=coupon rate, C=redemption, n=periods, yield i

Premium/discount bond formula

$P - C = (Fr - Ci)\, a_{\overline{n}|}$

Premium if $Fr > Ci$; discount if $Fr < Ci$

Makeham bond formula

$P = K + \frac{g}{i}(C - K)$

$K = Cv^n$, $g = Fr/C$ (modified coupon rate)

GENERAL CASH FLOWS, PORTFOLIOS, AND ASSET-LIABILITY MANAGEMENT

6 items

Macaulay duration

$D_{\text{Mac}} = \frac{\sum_t t\, v^t\, CF_t}{\sum_t v^t\, CF_t}$

Weighted average time of cash flows

Modified duration

$D_{\text{mod}} = \frac{D_{\text{Mac}}}{1+i}$

$\Delta P \approx -D_{\text{mod}}\, P\, \Delta i$

Convexity

$C = \frac{1}{P}\frac{d^2 P}{di^2}$

Price approximation: $\Delta P \approx -D_{\text{mod}}\, P\, \Delta i + \frac{1}{2} C\, P\, (\Delta i)^2$

Immunization (Redington) conditions

1. $PV_{\text{assets}} = PV_{\text{liabilities}}$
2. $D_{\text{assets}} = D_{\text{liabilities}}$ (durations match)
3. $C_{\text{assets}} > C_{\text{liabilities}}$ (convexity condition)

Forward interest rate

$(1+i_n)^n (1+f_{n,m})^m = (1+i_{n+m})^{n+m}$

$f_{n,m}$ = m-year forward rate starting in n years

Net present value

$NPV = \sum_{t=0}^{n} v^t\, CF_t$

Accept project if $NPV > 0$