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FORMULA SHEET

# Exam FAM

SOA · Fundamentals of Actuarial Math

23 FORMULAS

9 TOPICS

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## SEVERITY, FREQUENCY, AND AGGREGATE MODELS

7 items

### Compound distribution — mean

$E[S] = E[N] \cdot E[X]$

$S = X_1 + \cdots + X_N$, $N$=frequency, $X_i$=severity (iid, indep of $N$)

### Compound distribution — variance

$\mathrm{Var}(S) = E[N]\,\mathrm{Var}(X) + \mathrm{Var}(N)\,(E[X])^2$

### Compound Poisson — variance

$\mathrm{Var}(S) = \lambda\, E[X^2]$

(since $E[N] = \mathrm{Var}(N) = \lambda$)

### Stop-loss expected payment

$E[(S-d)_+] = E[S] - E[S \wedge d]$

$d$=aggregate retention

### Panjer recursion

$f_S(x) = \frac{1}{1 - a\, f_X(0)} \sum_{y=1}^{x} \left(a + \frac{b\,y}{x}\right) f_X(y)\, f_S(x-y)$

Applies to $(a, b, 0)$ frequency class

### Limited expected value — Pareto

$E[X \wedge u] = \frac{\theta}{\alpha - 1}\left[1 - \left(\frac{\theta}{u+\theta}\right)^{\alpha-1}\right], \quad \alpha > 1$

### Mean excess loss — Pareto

$e(d) = \frac{d+\theta}{\alpha - 1}$

Pareto: excess loss variable is also Pareto

## PARAMETRIC ESTIMATION

1 item

### MLE for exponential (\(\theta\) parameterization)

$\hat{\theta} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

(MLE=sample mean for exponential mean parameter)

## INTRODUCTION TO CREDIBILITY

2 items

### Buhlmann credibility estimate

$\hat{\mu} = Z\bar{X} + (1-Z)\mu_0$

$Z = \frac{n}{n+k}, \quad k = \frac{v}{a}$

$v = E[\sigma^2(\theta)],\ a = \mathrm{Var}(\mu(\theta))$

### Buhlmann-Straub credibility estimate

$Z = \frac{m}{m+k}, \quad k = \frac{v}{a}$

where $m = \sum_i m_i$ (total exposure)

## PRICING AND RESERVING FOR SHORT-TERM INSURANCE COVERAGES

1 item

### Bornhuetter-Ferguson ultimate loss

$U = \text{Actual Dev.} + \text{Expected Unreported}$

$= C + (1-q)\, ELR \cdot Premium$

$q$=reported fraction, $ELR$=expected loss ratio

## OPTION PRICING FUNDAMENTALS

3 items

### Black-Scholes call price

$C = S_0 N(d_1) - Ke^{-rT} N(d_2)$

$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$

### Black-Scholes put price

$P = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$

Same $d_1, d_2$ as call formula

### Put-call parity

$C - P = S_0 - Ke^{-rT}$

$C$=call, $P$=put, same $K$ and $T$

## LONG-TERM INSURANCE COVERAGES AND RETIREMENT FINANCIAL SECURITY PROGRAMS

2 items

### Curtate future lifetime — mean

$e_x = \sum_{k=1}^{\infty} {}_kp_x$

${}_kp_x = P(T_x > k)$

### Complete future lifetime — mean

$\mathring{e}_x = \int_0^{\infty} {}_tp_x \, dt$

## MORTALITY MODELS

1 item

### Survival function from force of mortality

${}_tp_x = \exp\left(-\int_0^t \mu_{x+s} \, ds\right)$

## PRESENT VALUE RANDOM VARIABLES FOR LONG-TERM INSURANCE COVERAGES

3 items

### Whole life insurance APV (discrete)

$A_x = \sum_{k=0}^{\infty} v^{k+1} \, {}_kp_x \, q_{x+k}$

### Variance of whole life insurance

$\mathrm{Var}(Z) = {}^2A_x - (A_x)^2$

${}^2A_x$ evaluated at $v^* = v^2$ (i.e., rate $i^* = (1+i)^2 - 1$)

### Term insurance APV (discrete, \(n\)-year)

$A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1} \, {}_kp_x \, q_{x+k}$

## PREMIUM AND POLICY VALUE CALCULATION FOR LONG-TERM INSURANCE COVERAGES

3 items

### Insurance-annuity relationship

$A_x = 1 - d \, \ddot{a}_x$

$\bar{A}_x = 1 - \delta \, \bar{a}_x$ (continuous)

### Net premium (benefit premium)

$P = \dfrac{A_x}{\ddot{a}_x}$

(equivalence principle: APV premiums = APV benefits)

### Prospective policy value

${}_tV = A_{x+t} - P \, \ddot{a}_{x+t}$

PV future benefits minus PV future premiums