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FORMULA SHEET

Exam ASTAM

SOA · Advanced Short-Term Actuarial Math

21 FORMULAS

6 TOPICS

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SEVERITY MODELS
3 items

Limited expected value — general formula
$E[X \wedge u] = \int_0^u [1 - F(x)]\,dx$
(for $X \ge 0$)

Limited expected value — Pareto
$$E[X \wedge u] = \frac{\theta}{\alpha - 1}\left[1 - \left(\frac{\theta}{u+\theta}\right)^{\alpha-1}\right], \quad \alpha > 1$$

Mean excess loss function
$$e(d) = E[X - d \mid X > d] = \frac{E[X] - E[X \wedge d]}{S(d)}$$

AGGREGATE MODELS
4 items

Compound distribution — mean and variance
$E[S] = E[N] \cdot E[X]$
$\mathrm{Var}(S) = E[N]\,\mathrm{Var}(X) + \mathrm{Var}(N)\,(E[X])^2$

Compound Poisson variance
$\mathrm{Var}(S) = \lambda\,E[X^2]$
(uses $E[N] = \mathrm{Var}(N) = \lambda$)

Stop-loss expected value
$E[(S - d)_+] = E[S] - E[S \wedge d]$

Panjer recursion
$$f_S(x) = \frac{1}{1 - a f_X(0)} \sum_{y=1}^{x} \left(a + \frac{by}{x}\right) f_X(y)\, f_S(x - y)$$
$(a, b, 0)$ class: Poisson $(0, \lambda)$, Binomial $(-p/(1-p),\ (n+1)p/(1-p))$

COVERAGE MODIFICATIONS
4 items

Payment per loss with ordinary deductible
$Y^L = (X - d)_+ = \max(X - d, 0)$
$E[Y^L] = E[X] - E[X \wedge d]$

Payment per payment (excess loss variable)
$Y^P = X - d \mid X > d$
$$E[Y^P] = e(d) = \frac{E[X] - E[X \wedge d]}{1 - F(d)}$$

Payment per loss with deductible \(d\) and limit \(u\)
$Y^L = \min(\max(X - d, 0),\ u)$
$E[Y^L] = E[X \wedge (d + u)] - E[X \wedge d]$

Loss elimination ratio
$$LER(d) = \frac{E[X \wedge d]}{E[X]}$$
Fraction of expected loss eliminated by deductible d

CONSTRUCTION AND SELECTION OF PARAMETRIC MODELS
3 items

MLE — likelihood and log-likelihood
$L(\theta) = \prod_{i=1}^{n} f(x_i; \theta)$
$\ell(\theta) = \sum_{i=1}^{n} \ln f(x_i; \theta)$
Solve $\frac{d\ell}{d\theta} = 0$

Chi-square goodness-of-fit statistic
$$\chi^2 = \sum_{j=1}^{k} \frac{(O_j - E_j)^2}{E_j}$$
df = cells − 1 − estimated parameters

Kolmogorov-Smirnov test statistic
$D = \sup_x |F_n(x) - F(x; \hat\theta)|$
F_n=empirical CDF; reject H_0 if D exceeds critical value

CREDIBILITY
3 items

Buhlmann credibility estimate
$\hat\mu = Z\bar X + (1 - Z)\mu_0$
$Z = \frac{n}{n+k}, \quad k = \frac{v}{a}$
$v = E[\sigma^2(\theta)],\ a = \mathrm{Var}(\mu(\theta))$

Buhlmann-Straub credibility
$Z = \frac{m}{m+k}, \quad k = \frac{v}{a}, \quad m = \sum_i m_i$
Weighted by exposures m_i

Empirical Bayes — \(v\) and \(a\) estimates
$$\hat v = \frac{1}{r(n-1)} \sum_{i=1}^{r} \sum_{j=1}^{n} (X_{ij} - \bar X_i)^2$$
$\hat a = \frac{\hat v(n\hat a^* - v)}{\cdots}$ (use unbiased moment estimators from Buhlmann-Straub setup)

Chain-ladder (development) method

$\hat{C}_{i,k+1} = \hat{f}_k \cdot C_{i,k}$

$\hat{f}_k = \frac{\sum_i C_{i,k+1}}{\sum_i C_{i,k}}$ (volume-weighted average development factor)

Bornhuetter-Ferguson ultimate loss

$\hat{U}_i = C_{i,\text{current}} + (1 - q_i) \cdot ELR \cdot P_i$

q_i=\% reported, P_i=premium, ELR=expected loss ratio

IBNR reserve

$IBNR = \hat{U} - C_{\text{current}}$

(estimated ultimate minus cumulative paid/reported losses)

Loss ratio

$LR = \frac{\text{Losses Incurred}}{\text{Earned Premium}}$