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FORMULA SHEET

Exam ALTAM

SOA · Advanced Long-Term Actuarial Math

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FORMULAS

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TOPICS

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SURVIVAL MODELS FOR CONTINGENT CASH FLOWS

5 items

Survival function from force of mortality

$${}_tp_x = \exp\left(-\int_0^t \mu_{x+s}\,ds\right)$$

Force of mortality — definition

$$\mu_{x+t} = -\frac{d}{dt}\ln {}_tp_x = \frac{f_T(t)}{{}_tp_x}$$

Makeham force of mortality

$$\mu_x = A + Bc^x, \quad A \geq 0,\ B > 0,\ c > 1$$

A = accident hazard, Bc^x = aging hazard

Complete future lifetime — mean

$$\dot{e}_x = \int_0^\infty {}_tp_x\,dt$$

Curtate future lifetime — mean

$$e_x = \sum_{k=1}^\infty {}_kp_x$$

PREMIUM AND POLICY VALUATION FOR LONG-TERM COVERAGES

8 items

Whole life insurance APV (continuous)

$$\bar{A}_x = \int_0^\infty e^{-\delta t}\,{}_tp_x\,\mu_{x+t}\,dt$$

Endowment insurance APV (discrete)

$$A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + v^n\,{}_np_x$$

= term + pure endowment

Life annuity-due APV

$$\ddot{a}_x = \sum_{k=0}^\infty v^k\,{}_kp_x = \frac{1 - A_x}{d}$$

Insurance-annuity relationship

$$A_x = 1 - d\,\ddot{a}_x$$

$$\bar{A}_x = 1 - \delta\,\bar{a}_x \text{ (continuous)}$$

Net premium (benefit equivalence)

$$P = \frac{A_x}{\ddot{a}_x} \quad \text{(whole life, discrete)}$$

Prospective policy value

$${}_tV = A_{x+t} - P\,\ddot{a}_{x+t}$$

Recursive policy value

$$({}_tV + P)(1+i) = q_{x+t}\cdot b_{t+1} + p_{x+t}\cdot {}_{t+1}V$$

b_{t+1}=death benefit in year $t+1$

Thiele's differential equation

$$\frac{d}{dt}({}_t\bar{V}) = \delta\,{}_t\bar{V} + P_t - \mu_{x+t}(b_t - {}_t\bar{V})$$

P_t=premium rate, b_t=death benefit at time t

JOINT LIFE INSURANCE AND ANNUITIES

3 items

Joint life survival function

$${}_tp_{xy} = {}_tp_x \cdot {}_tp_y \text{ (if independent)}$$

Joint life status fails at first death

Last survivor survival function

$${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy}$$

Joint life annuity-due APV

$$\ddot{a}_{xy} = \sum_{k=0}^\infty v^k\,{}_kp_{xy}$$

PROFIT ANALYSIS

2 items

Profit signature

$$\Pi_t = ({}_{t-1}V + P_t - e_t)(1+i_t) - q_{x+t-1}\cdot b_t - p_{x+t-1}\cdot {}_tV$$

$t = 1, 2, \ldots$

NPV of profits

$$NPV = \sum_{t=0}^n v_h^t\,{}_tp_x\,\Pi_t$$

v_h=discount at hurdle rate; Π_0=issue expense

PENSION PLANS AND RETIREMENT BENEFITS

1 item

DB pension — projected unit credit (PUC) liability

$$AL_t = b_t \cdot \ddot{a}^{\text{ret}}_{x+t} \cdot p^{\text{retire}}_x \cdot v^{r-t}$$

b_t=accrued benefit at time t

UNIVERSAL LIFE INSURANCE

2 items

UL account value recursion

$$AV_t = (AV_{t-1} + P_t - e_t - COI_t)(1 + i_t^{\text{credit}})$$

COI_t=cost of insurance charge

UL cost of insurance

$$COI_t = v\, q_{x+t-1} \cdot (b_t - AV_t)$$

(net amount at risk $\times$ discounted q)