FreeFellow

FORMULA SHEET

CFA Level III: Private Wealth

Chartered Financial Analyst

29
FORMULAS

7
TOPICS

freefellow.org/cfa-level3-pw/formulas

WEALTH PLANNING

5 items

Human capital present value

$HC_0 = \sum_{t=1}^{N} \frac{w_t \cdot (1-p_{death,t})}{(1+r)^t}$

w_t=expected earnings, r=discount rate.

Total wealth = financial + human capital.

Taxable account accumulation

$FV = [1 + r(1-t)]^n$

Annual returns taxed each year; interest/dividend taxed as ordinary income

For deferred capital gains: $FV = (1+r)^n - [(1+r)^n - 1] \times t_{CG}$

Tax-deferred account accumulation

$FV = (1+r)^n(1-T_n)$

T_n = tax rate at withdrawal

Contributions pre-tax; all withdrawals taxed as ordinary income

Favorable when future tax rate < current rate

Tax-exempt account accumulation

$FV = (1+r)^n$

After-tax contributions; all growth and withdrawals tax-free

Favorable when future tax rate > current rate

Roth IRA / Roth 401(k) structure

Relative advantage of tax-deferred vs taxable

$RA = \frac{(1+r)^n(1-T_n)}{[1+r(1-t)]^n(1-T_0)}$

RA > 1 → tax-deferred preferred

T_n = future tax rate, T_0 = current tax rate

Higher returns and longer horizons amplify advantage

INVESTMENT PLANNING

4 items

After-tax return

$r_{AT} = r_{PT} \times (1-t)$

For return with mixed income types:

$r_{AT} = r_{CG}(1-t_{CG}) + r_{Inc}(1-t_{Inc}) + r_{tax-free}$

r_PT = pre-tax return, t = tax rate

Core capital estimate

$CC = \sum_{t=1}^{T} \frac{E_t}{(1+r_{AT})^t} \times p_{survive,t}$

E_t = expenses in year t, r_AT = after-tax return

p_survive,t = probability of surviving to year t

Core capital must be preserved; surplus is investable

Tax-loss harvesting benefit

$\text{Benefit} = t_{ST} \cdot L - t_{LT} \cdot G_f$

L=loss harvested, G_f=future gain at lower basis.

Net positive when $t_{ST} > t_{LT}$ or long horizon.

Asset location decision

Tax-inefficient assets (bonds, REITs, high-turnover funds) → tax-deferred accounts

Tax-efficient assets (equities, index funds, muni bonds) → taxable accounts

Rationale: maximize after-tax wealth across entire household balance sheet

TRANSFERRING THE WEALTH

3 items

Estate tax (simple)

$\text{Tax} = (\text{Estate value} - \text{Exemption}) \times t_e$

t_e = estate tax rate

Taxable estate = gross estate - debts - marital deduction - charitable deduction

Relative value of gift vs bequest

$RV = \frac{(1+r_g)^n(1-T_e)}{(1+r_e)^n(1-T_g)+T_g-T_e}$

r_g = after-tax return if gifted, r_e = after-tax return if bequested

T_e = estate/bequest tax rate, T_g = gift tax rate

RV > 1 → gifting preferred

Generation-Skipping Transfer (GST) tax

Applies to transfers skipping a generation (grandparent → grandchild).

$\text{GST tax} = \text{Transfer amount} \times t_{GST}$

Separate exemption; dynasty trusts avoid multi-gen estate tax.

TOPIC 1

4 items

Mean-variance optimal portfolio weight

$\mathbf{w}^* = \frac{1}{\lambda}\Sigma^{-1}(\mu - r_f\mathbf{1})$

λ = risk aversion, Σ = covariance matrix, μ = expected returns

Corner portfolio blending

$w_A = \frac{E(R_P)-E(R_B)}{E(R_A)-E(R_B)}$, $w_B = 1 - w_A$

Blend two adjacent corner portfolios A and B to achieve target return E(R_P)

All blends lie on the efficient frontier

Black-Litterman expected return

Equilibrium: $\Pi = \delta\Sigma w_{mkt}$

Blended: $E(R) = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$

δ=risk aversion, w_{mkt}=mkt cap weights

Portfolio rebalancing trigger (range-based)

Rebalance when: $|w_i - w_i^*| > \Delta_i$

w_i^* = target weight, Δ_i = tolerance band

Wider bands → lower costs, less precision

Correlation-adjusted bands: wider for high-correlation assets

TOPIC 2

3 items

Marginal Contribution to Risk (MCTR)

$MCTR_i = \beta_i \times \sigma_p$

$\beta_i = \frac{\text{Cov}(R_i,R_p)}{\sigma_p^2}$

Measures risk added by a small increase in asset i's weight

Absolute Contribution to Risk (ACTR)

$ACTR_i = w_i \times MCTR_i = w_i \times \beta_i \times \sigma_p$

$\sum_i ACTR_i = \sigma_p$ (contributions sum to total portfolio risk)

Risk budget = set target ACTRs

Tracking error

$TE = \sigma(R_p - R_B) = \sqrt{\frac{\sum(r_{p,t}-r_{B,t}-\bar{\alpha})^2}{T-1}}$

Also called active risk or tracking risk

Annualized: $TE_{annual} = TE_{monthly} \times \sqrt{12}$

TOPIC 3

5 items

Information ratio

$IR = \frac{\bar{R}_p - \bar{R}_B}{\sigma(R_p - R_B)} = \frac{\bar{\alpha}}{TE}$

$\bar{\alpha}$ = mean active return, TE = tracking error

Measures active return per unit of active risk

Fundamental Law of Active Management

$IR = IC \times \sqrt{BR}$

IC = information coefficient, BR = investment breadth

Expected active return: $E(R_A) = IC \times \sqrt{BR} \times \sigma_A$ (TC assumed = 1)

BHB attribution effects

Allocation: $(w_{p,i} - w_{B,i})(R_{B,i} - R_B)$

Selection: $w_{B,i}(R_{p,i} - R_{B,i})$

Interaction: $(w_{p,i} - w_{B,i})(R_{p,i} - R_{B,i})$

Sum of all effects = total active return

Sharpe ratio

$SR_p = \frac{R_p - R_f}{\sigma_p}$

Reward-to-variability ratio using total risk

Sharpe of combined portfolio: $SR_C^2 = SR_B^2 + IR^2$

M-squared (M²)

$M^2 = (R_p - R_f)\frac{\sigma_m}{\sigma_p} + R_f$

Risk-adjusted return scaled to match market's volatility

M² > R_m → portfolio outperformed on risk-adjusted basis

Delta of call and put

Call: $\Delta_c = N(d_1) \in (0, 1)$

Put: $\Delta_p = N(d_1) - 1 \in (-1, 0)$

Put-call: $\Delta_c - \Delta_p = 1$

Approx change in option price for \$1 change in underlying

Protective put payoff

At expiration: $\text{Payoff} = S_T + \max(X - S_T, 0)$

= $\max(S_T, X)$

Profit = Payoff - (S_0 + p), where p = put premium

Limits downside while preserving upside

Collar payoff at expiration

Long stock + long put (X_L) + short call (X_H)

Payoff: $S_T + \max(X_L - S_T, 0) - \max(S_T - X_H, 0)$

= $\min(\max(S_T, X_L), X_H)$

Limits gains above X_H, protects below X_L

Covered call payoff at expiration

Long stock + short call (X)

Payoff: $S_T - \max(S_T - X, 0) = \min(S_T, X)$

Profit = Payoff - S_0 + c (c = call premium received)

Caps upside; enhances income in flat/down markets

Variance swap payoff

$\text{Payoff} = N_{vega} \times (\sigma^2_{realized} - \sigma^2_{strike})$

Long variance swap profits when realized variance > strike variance

No delta-hedging needed; pure volatility exposure