FreeFellow

FORMULA SHEET

CFA Level III: Private Markets

Chartered Financial Analyst

31 FORMULAS

7 TOPICS

freefellow.org/cfa-level3-pmkt/formulas

GP & INVESTOR PERSPECTIVES
4 items

DPI (Distributed to Paid-In)
$DPI = \frac{\text{Cumulative distributions}}{\text{Paid-in capital}}$

DPI < 1 → still in J-curve; DPI > 1 → returned capital with profit

Purely realized metric; key for LP liquidity analysis

RVPI (Residual Value to Paid-In)
$RVPI = \frac{\text{Residual NAV}}{\text{Paid-in capital}}$

Unrealized portion of TVPI

Declining RVPI over fund life signals realization progress

TVPI = DPI + RVPI

J-curve
Early: negative CFs (calls + fees, no distributions) → negative/low net IRR

Inflection: distributions exceed contributions

Steeper = faster deployment + value creation

Management fee base
Investment period: fees on committed capital.

Post-investment: fees on invested (deployed) capital.

$\text{Fee} = \text{Rate} \times \text{Base}$ (typically 1.5–2%)

PRIVATE EQUITY
4 items

MOIC (Multiple on Invested Capital)
$MOIC = \frac{\text{Distributions+Residual NAV}}{\text{Invested capital}}$

Also called TVPI. TVPI = DPI + RVPI.

IRR for private equity
Solve for r in: $\sum_{t=0}^{T} \frac{CF_t}{(1+r)^t} = 0$

CF_0 = -initial investment (negative), CF_T includes terminal NAV

Gross IRR = fund-level; Net IRR = LP-level (after fees/carry)

Levered equity return
$r_e = r_u + \frac{D}{E}(r_u - r_d)$

r_u = unlevered asset return, r_d = cost of debt

D/E = debt-to-equity ratio

Leverage amplifies equity returns (and risk)

Waterfall distribution (PE)
Order: Return of capital → Preferred return (hurdle) → GP catch-up → Carry split

American: deal-by-deal. European: whole-fund before carry.

Typical: 20% carry above 8% hurdle.

PRIVATE REAL ESTATE
6 items

Loan-to-Value (LTV)
$LTV = \frac{\text{Loan amount}}{\text{Property value}}$

Higher LTV → higher leverage, more risk

Typical real estate: 60–75% LTV

Lenders covenant on LTV and DSCR

Capitalization rate (Cap rate)
$r_{cap} = \frac{NOI}{V}$

NOI = net operating income (stabilized, before debt service)

V = property value

Inverse: $V = \frac{NOI}{r_{cap}}$

Lower cap rate = higher valuation multiple

Net Operating Income (NOI)
$NOI = \text{EGI} - \text{Operating expenses}$

EGI = Potential gross income − Vacancy & credit losses

Opex excludes debt service, depreciation, income taxes.

Direct capitalization value
$V = \frac{NOI}{r_{cap}}$

Stabilized NOI used (normalized for occupancy, expenses)

Cap rate sourced from comparable sales

Used for income-producing properties

Debt Service Coverage Ratio (DSCR)
$DSCR = \frac{NOI}{\text{Annual debt service}}$

Annual debt service = principal + interest payments

DSCR > 1 → property generates sufficient income to cover debt

Lenders typically require DSCR ≥ 1.2×

NAV per unit (private real estate fund)
$NAV = \text{Appraised values} + \text{Other assets} - \text{Liabilities}$

$NAV \text{ per unit} = \frac{NAV}{\text{Units outstanding}}$

Appraisals quarterly; lags public pricing.

TOPIC 1

4 items

Mean-variance optimal portfolio weight

$\mathbf{w}^* = \frac{1}{\lambda}\Sigma^{-1}(\mu - r_f\mathbf{1})$

λ = risk aversion, Σ = covariance matrix, μ = expected returns

Corner portfolio blending

$w_A = \frac{E(R_P)-E(R_B)}{E(R_A)-E(R_B)}$, $w_B = 1 - w_A$

Blend two adjacent corner portfolios A and B to achieve target return E(R_P)

All blends lie on the efficient frontier

Black-Litterman expected return

Equilibrium: $\Pi = \delta\Sigma w_{mkt}$

Blended: $E(R) = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$

δ=risk aversion, w_{mkt}=mkt cap weights

Portfolio rebalancing trigger (range-based)

Rebalance when: $|w_i - w_i^*| > \Delta_i$

w_i^* = target weight, Δ_i = tolerance band

Wider bands → lower costs, less precision

Correlation-adjusted bands: wider for high-correlation assets

TOPIC 2

3 items

Marginal Contribution to Risk (MCTR)

$MCTR_i = \beta_i \times \sigma_p$

$\beta_i = \frac{\text{Cov}(R_i,R_p)}{\sigma_p^2}$

Measures risk added by a small increase in asset i's weight

Absolute Contribution to Risk (ACTR)

$ACTR_i = w_i \times MCTR_i = w_i \times \beta_i \times \sigma_p$

$\sum_i ACTR_i = \sigma_p$ (contributions sum to total portfolio risk)

Risk budget = set target ACTRs

Tracking error

$TE = \sigma(R_p - R_B) = \sqrt{\frac{\sum(r_{p,t}-r_{B,t}-\bar{\alpha})^2}{T-1}}$

Also called active risk or tracking risk

Annualized: $TE_{annual} = TE_{monthly} \times \sqrt{12}$

TOPIC 3

5 items

Information ratio

$IR = \frac{\bar{R}_p - \bar{R}_B}{\sigma(R_p - R_B)} = \frac{\bar{\alpha}}{TE}$

$\bar{\alpha}$ = mean active return, TE = tracking error

Measures active return per unit of active risk

Fundamental Law of Active Management

$IR = IC \times \sqrt{BR}$

IC = information coefficient, BR = investment breadth

Expected active return: $E(R_A) = IC \times \sqrt{BR} \times \sigma_A$ (TC assumed = 1)

BHB attribution effects

Allocation: $(w_{p,i} - w_{B,i})(R_{B,i} - R_B)$

Selection: $w_{B,i}(R_{p,i} - R_{B,i})$

Interaction: $(w_{p,i} - w_{B,i})(R_{p,i} - R_{B,i})$

Sum of all effects = total active return

Sharpe ratio

$SR_p = \frac{R_p - R_f}{\sigma_p}$

Reward-to-variability ratio using total risk

Sharpe of combined portfolio: $SR_C^2 = SR_B^2 + IR^2$

M-squared (M²)

$M^2 = (R_p - R_f)\frac{\sigma_m}{\sigma_p} + R_f$

Risk-adjusted return scaled to match market's volatility

M² > R_m → portfolio outperformed on risk-adjusted basis

TOPIC 4

5 items

Delta of call and put

Call: $\Delta_c = N(d_1) \in (0,1)$

Put: $\Delta_p = N(d_1) - 1 \in (-1,0)$

Put-call: $\Delta_c - \Delta_p = 1$

Approx change in option price for \$1 change in underlying

Protective put payoff

At expiration: $\text{Payoff} = S_T + \max(X - S_T, 0)$

= $\max(S_T, X)$

Profit = Payoff - (S_0 + p), where p = put premium

Limits downside while preserving upside

Collar payoff at expiration

Long stock + long put (X_L) + short call (X_H)

Payoff: $S_T + \max(X_L - S_T, 0) - \max(S_T - X_H, 0)$

= $\min(\max(S_T, X_L), X_H)$

Limits gains above X_H, protects below X_L

Covered call payoff at expiration

Long stock + short call (X)

Payoff: $S_T - \max(S_T - X, 0) = \min(S_T, X)$

Profit = Payoff - S_0 + c (c = call premium received)

Caps upside; enhances income in flat/down markets

Variance swap payoff

$\text{Payoff} = N_{vega} \times (\sigma^2_{realized} - \sigma^2_{strike})$

Long variance swap profits when realized variance > strike variance

No delta-hedging needed; pure volatility exposure