

FREEFELLOW

FORMULA SHEET

CFA LEVEL III: PORTFOLIO MANAGEMENT

Chartered Financial Analyst

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FORMULAS

11

TOPICS

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INDEX-BASED EQUITY STRATEGIES

1 item

Tracking error decomposition

$$TE^2 = TE_{factor}^2 + TE_{specific}^2$$

Factor TE from active factor exposures × factor covariance matrix

Specific TE from active stock-specific bets

Index funds minimize both; active strategies accept higher TE

ACTIVE EQUITY INVESTING

2 items

Jensen's alpha

$$\alpha = R_p - [R_f + \beta_p(R_m - R_f)]$$

CAPM-adjusted outperformance

Fundamental law link: $E(\alpha) = IC \times \sqrt{BR} \times \sigma_A \times TC$

Treyner ratio

$$T_p = \frac{R_p - R_f}{\beta_p}$$

Excess return per unit of systematic (beta) risk

Use when portfolio is one component of a larger portfolio

LIABILITY-DRIVEN & INDEX-BASED STRATEGIES

3 items

DV01 (Dollar Value of 01)

$$DV01 = \frac{\text{Modified duration} \times V}{10000}$$

Dollar price change for a 1 bp decline in yield

V = portfolio market value

Futures contracts for duration adjustment

$$N_f = \frac{(D_{target} - D_{portfolio}) \times V_{portfolio}}{D_{futures} \times V_{futures}}$$

D = duration, V = dollar value

Positive N_f → buy futures (extend duration)

Negative N_f → sell futures (shorten duration)

Liability-driven surplus duration

$$\Delta \text{Surplus} = (D_A \times A - D_L \times L) \times \Delta y$$

D_A = asset duration, A = asset value

D_L = liability duration, L = liability value

Immunize when: $D_A \times A = D_L \times L$

YIELD CURVE STRATEGIES

3 items

Key rate duration

$$\% \Delta P \approx - \sum_k KRD_k \times \Delta y_k$$

KRD_k = key rate duration at maturity k

Used for non-parallel yield curve shifts; sum of KRDs ≈ effective duration

Butterfly spread conditions

Net position: long belly, short wings (or reverse)

Positive butterfly = yield curve humps in the middle

Profit condition (long butterfly): $2y_{5Y} < y_{2Y} + y_{10Y}$

Measures curvature of yield curve

Barbell vs bullet convexity

Barbell (short + long): higher convexity

Bullet (middle): lower convexity

At same duration: $C_{barbell} > C_{bullet}$

Barbell wins in high vol; bullet in stable curves

CREDIT STRATEGIES

2 items

Credit spread duration

$$\% \Delta P \approx - \text{Spread duration} \times \Delta s$$

Δs = change in credit spread

For corporate bonds: spread duration ≈ modified duration

For floating-rate notes: spread duration ≈ time to reset

Excess return over Treasuries

$$XR \approx s \cdot t - \Delta s \cdot D_s - t \cdot p \cdot L$$

s=spread, t=horizon, D_s=spread duration, p=PD, L=loss rate

Implementation shortfall

$$IS = \frac{\text{Paper portfolio gain} - \text{Actual portfolio gain}}{\text{Investment decision value}}$$

Components: delay cost + trading cost + opportunity cost

Measures total cost of executing a trade vs decision price

VWAP benchmark

$$VWAP = \frac{\sum_t (P_t \times V_t)}{\sum_t V_t}$$

P_t = price at time t, V_t = volume at time t

Trade cost vs VWAP = (VWAP - execution price) for buys

Limitation: manipulable; meaningless for large orders

Market impact cost

Market impact = $(P_{exec} - P_{pre}) / P_{pre}$ (for buys)

P_{exec} = average execution price, P_{pre} = pre-trade benchmark

Temporary impact reverses; permanent impact does not

Higher urgency → more market impact

CASE STUDY: ENDOWMENT

1 item

DV01-based hedge ratio

$$N = -\frac{DV01_{portfolio}}{DV01_{hedge}}$$

Negative = short the hedge instrument

For cross-hedge: $N = -\frac{DV01_p}{DV01_h} \times \beta_{spread}$

TOPIC 1

4 items

Mean-variance optimal portfolio weight

$$\mathbf{w}^* = \frac{1}{\lambda} \Sigma^{-1} (\boldsymbol{\mu} - r_f \mathbf{1})$$

λ = risk aversion, Σ = covariance matrix, μ = expected returns

Corner portfolio blending

$$w_A = \frac{E(R_P) - E(R_B)}{E(R_A) - E(R_B)}, w_B = 1 - w_A$$

Blend two adjacent corner portfolios A and B to achieve target return E(R_P)

All blends lie on the efficient frontier

Black-Litterman expected return

Equilibrium: $\Pi = \delta \Sigma w_{mkt}$

Blended: $E(R) = [(\tau \Sigma)^{-1} + P^T \Omega^{-1} P]^{-1} [(\tau \Sigma)^{-1} \Pi + P^T \Omega^{-1} Q]$

δ = risk aversion, w_{mkt} = mkt cap weights

Portfolio rebalancing trigger (range-based)

Rebalance when: $|w_i - w_i^*| > \Delta_i$

w_i^* = target weight, Δ_i = tolerance band

Wider bands → lower costs, less precision

Correlation-adjusted bands: wider for high-correlation assets

TOPIC 2

3 items

Marginal Contribution to Risk (MCTR)

$$MCTR_i = \beta_i \times \sigma_p$$

$$\beta_i = \frac{\text{Cov}(R_i, R_p)}{\sigma_p^2}$$

Measures risk added by a small increase in asset i's weight

Absolute Contribution to Risk (ACTR)

$$ACTR_i = w_i \times MCTR_i = w_i \times \beta_i \times \sigma_p$$

$\sum_i ACTR_i = \sigma_p$ (contributions sum to total portfolio risk)

Risk budget = set target ACTRs

Tracking error

$$TE = \sigma(R_p - R_B) = \sqrt{\frac{\sum (r_{p,t} - r_{B,t} - \bar{\alpha})^2}{T-1}}$$

Also called active risk or tracking risk

Annualized: $TE_{annual} = TE_{monthly} \times \sqrt{12}$

Information ratio

$$IR = \frac{\bar{R}_p - \bar{R}_B}{\sigma(\bar{R}_p - \bar{R}_B)} = \frac{\bar{\alpha}}{TE}$$

$\bar{\alpha}$ = mean active return, TE = tracking error

Measures active return per unit of active risk

Fundamental Law of Active Management

$$IR = IC \times \sqrt{BR}$$

IC = information coefficient, BR = investment breadth

Expected active return: $E(R_A) = IC \times \sqrt{BR} \times \sigma_A$ (TC assumed = 1)

BHB attribution effects

Allocation: $(w_{p,i} - w_{B,i})(R_{B,i} - R_B)$

Selection: $w_{B,i}(R_{p,i} - R_{B,i})$

Interaction: $(w_{p,i} - w_{B,i})(R_{p,i} - R_{B,i})$

Sum of all effects = total active return

Sharpe ratio

$$SR_p = \frac{R_p - R_f}{\sigma_p}$$

Reward-to-variability ratio using total risk

Sharpe of combined portfolio: $SR_C^2 = SR_B^2 + IR^2$

M-squared (M²)

$$M^2 = (R_p - R_f) \frac{\sigma_m}{\sigma_p} + R_f$$

Risk-adjusted return scaled to match market's volatility

$M^2 > R_m \rightarrow$ portfolio outperformed on risk-adjusted basis

TOPIC 4

Delta of call and put

Call: $\Delta_c = N(d_1) \in (0, 1)$

Put: $\Delta_p = N(d_1) - 1 \in (-1, 0)$

Put-call: $\Delta_c - \Delta_p = 1$

Approx change in option price for \$1 change in underlying

Protective put payoff

At expiration: Payoff = $S_T + \max(X - S_T, 0)$

= $\max(S_T, X)$

Profit = Payoff - ($S_0 + p$), where p = put premium

Limits downside while preserving upside

Collar payoff at expiration

Long stock + long put (X_L) + short call (X_H)

Payoff: $S_T + \max(X_L - S_T, 0) - \max(S_T - X_H, 0)$

= $\min(\max(S_T, X_L), X_H)$

Limits gains above X_H , protects below X_L

Covered call payoff at expiration

Long stock + short call (X)

Payoff: $S_T - \max(S_T - X, 0) = \min(S_T, X)$

Profit = Payoff - $S_0 + c$ (c = call premium received)

Caps upside; enhances income in flat/down markets

Variance swap payoff

$$\text{Payoff} = N_{vega} \times (\sigma_{realized}^2 - \sigma_{strike}^2)$$

Long variance swap profits when realized variance > strike variance

No delta-hedging needed; pure volatility exposure