

FREEFELLOW

FORMULA SHEET

CFA LEVEL I

Chartered Financial Analyst

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FORMULAS

9

TOPICS

freefellow.org/cfa-level1/formulas

QUANTITATIVE METHODS

10 items

Correlation coefficient

$$\rho_{XY} = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$$

Ranges from -1 to +1; unitless measure of linear association

Holding Period Return (HPR)

$$HPR = \frac{P_1 - P_0 + D_1}{P_0}$$

P₁ = ending price, P₀ = beginning price, D₁ = cash distributions received

Population variance

$$\sigma^2 = \frac{\sum_{i=1}^N (X_i - \mu)^2}{N}$$

μ = population mean, N = population size

Future Value of ordinary annuity

$$FV = PMT \times \frac{(1+r)^n - 1}{r}$$

PMT = periodic payment, r = periodic rate, n = periods

Present Value (single sum)

$$PV = \frac{FV}{(1+r)^n}$$

FV = future value, r = periodic rate, n = number of periods

Sharpe ratio

$$S_p = \frac{R_p - R_f}{\sigma_p}$$

R_p = portfolio return, R_f = risk-free rate, σ_p = portfolio std dev

Excess return per unit of total risk

Present Value of perpetuity

$$PV = \frac{PMT}{r}$$

PMT = periodic payment, r = discount rate

Sample variance

$$s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$

\bar{X} = sample mean, n = sample size (uses n-1 for unbiasedness)

Present Value of ordinary annuity

$$PV = PMT \times \frac{1 - (1+r)^{-n}}{r}$$

PMT = periodic payment, r = periodic rate, n = periods

Bayes' Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Updates prior probability P(A) given new information B

$$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$$

ECONOMICS

6 items

GDP expenditure approach

$$GDP = C + I + G + (X - M)$$

C = consumption, I = investment, G = government spending

X = exports, M = imports, (X-M) = net exports

Fisher effect

$$(1 + r_{nom}) = (1 + r_{real})(1 + \pi)$$

Approx: $r_{nom} \approx r_{real} + \pi$

r_{nom} = nominal rate, r_{real} = real rate, π = expected inflation

Money multiplier

$$m = \frac{1}{\text{reserve requirement}}$$

Maximum deposit expansion from a given reserve base

$$\Delta \text{Money supply} = m \times \Delta \text{Reserves}$$

Price elasticity of demand

$$E_d = \frac{\% \Delta Q_d}{\% \Delta P}$$

$|E_d| > 1$: elastic (revenue rises when P falls). $|E_d| < 1$: inelastic. = 1: unit-elastic (revenue maximized).

Breakeven and shutdown points

Breakeven: $P = ATC$ (price covers all costs).

Shutdown (short run): $P < AVC$. If $AVC < P < ATC$, keep operating short-run because contribution covers some fixed cost.

Cross-rate calculation

$$\frac{A}{C} = \frac{A}{B} \times \frac{B}{C}$$

Multiply or divide quoted rates to derive a cross-rate. Bid-ask: use bid for one leg and ask for the other to be conservative.

WACC

$$WACC = w_d r_d (1 - t) + w_p r_p + w_e r_e$$

w = weights (market value), r = required returns

d = debt, p = preferred, e = equity, t = tax rate

FCFF

$$FCFF = NI + NCC + Int(1 - t) - \Delta WC - CapEx$$

NI = net income, NCC = non-cash charges, Int = interest expense

ΔWC = change in working capital, t = tax rate

Free Cash Flow to Equity (FCFE)

$$FCFE = FCFF - Int(1 - t) + \Delta Debt$$

Cash available to equity holders after all obligations and reinvestment

Degree of Total Leverage (DTL)

$$DTL = DOL \times DFL = \frac{Q(P-V)}{Q(P-V)-F-I}$$

% change in EPS per 1% change in sales

Degree of Financial Leverage (DFL)

$$DFL = \frac{EBIT}{EBIT-I}$$

I = interest expense

% change in EPS per 1% change in EBIT

Degree of Operating Leverage (DOL)

$$DOL = \frac{Q(P-V)}{Q(P-V)-F} = \frac{\text{Contribution margin}}{EBIT}$$

Q = units, P = price, V = variable cost/unit, F = fixed costs

FINANCIAL STATEMENT ANALYSIS

Receivables turnover and DSO

$$\text{Receivables turnover} = \frac{\text{Revenue}}{\text{Avg Accounts Receivable}}$$

$$DSO = \frac{365}{\text{Receivables turnover}}$$

Days Sales Outstanding – average collection period

Current ratio

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

Measures short-term liquidity; higher = more liquid

Inventory turnover

$$\text{Inventory turnover} = \frac{\text{COGS}}{\text{Average inventory}}$$

$$\text{Days on hand (DOH): } DOH = \frac{365}{\text{Inventory turnover}}$$

Return on Equity (ROE)

$$ROE = \frac{\text{Net income}}{\text{Avg total equity}}$$

DuPont: $ROE = \text{Net margin} \times \text{Asset turnover} \times \text{Leverage}$. Drives sustainable growth: $g = b \times ROE$.

Return on Assets (ROA)

$$ROA = \frac{\text{Net income}}{\text{Average total assets}}$$

Alternative: $ROA = \text{Net profit margin} \times \text{Asset turnover}$

2-factor DuPont decomposition

3-factor DuPont decomposition

$$ROE = \frac{NI}{\text{Sales}} \times \frac{\text{Sales}}{\text{Assets}} \times \frac{\text{Assets}}{\text{Equity}}$$

Net profit margin \times Asset turnover \times Financial leverage

EQUITY INVESTMENTS

Justified P/E (leading)

$$\frac{P_0}{E_1} = \frac{1-b}{r-g}$$

b = retention ratio (1-b = payout ratio), r = required return, g = $ROE \times b$

Gordon Growth Model (DDM)

$$V_0 = \frac{D_1}{r-g} = \frac{D_0(1+g)}{r-g}$$

D₁ = next dividend, r = required return, g = constant growth rate

Requires $r > g$

Enterprise Value (EV)

$$EV = \text{Market cap} + \text{Debt} + \text{Preferred} - \text{Cash}$$

$EV/EBITDA$ = enterprise value multiple

Capital-structure-neutral valuation metric

Price-to-Book ratio

$$P/B = \frac{\text{Market price per share}}{\text{Book value per share}}$$

$$\text{Justified P/B: } \frac{ROE-g}{r-g}$$

$P/B > 1$ implies market values assets above book

Sustainable growth rate

$$g = b \times ROE$$

b = retention rate (1 - dividend payout). Maximum growth a firm can sustain without external financing.

P/E ratio (trailing & leading)

Trailing: $\frac{P_0}{EPS_0}$ – uses last 12 months EPS.

Leading: $\frac{P_0}{EPS_1}$ – uses next 12 months / forecast EPS. Forward-looking variant.

Forward rate from spot rates

$$(1 + z_2)^2 = (1 + z_1)(1 + {}_1f_1)$$

$$\text{General: } (1 + z_n)^n = (1 + z_{n-1})^{n-1}(1 + {}_{n-1}f_1)$$

z = spot rate, f = implied forward rate

Bond price

$$P = \sum_{t=1}^n \frac{C}{(1+r)^t} + \frac{FV}{(1+r)^n}$$

C = coupon payment, r = periodic YTM, n = periods, FV = face value

Macaulay duration

$$D_{Mac} = \frac{\sum_{t=1}^n t \cdot \frac{CF_t}{(1+r)^t}}{P}$$

Weighted average time to receive cash flows; measured in years

Modified duration

$$D_{Mod} = \frac{D_{Mac}}{1+r}$$

$$\% \Delta P \approx -D_{Mod} \times \Delta y$$

r = periodic YTM, Δy = change in yield

Current yield

$$\text{Current yield} = \frac{\text{Annual coupon}}{\text{Price}}$$

Simplest yield measure; ignores capital gains/losses and time value

Price value of a basis point (PVBP)

$$PVBP = |P_{y+0.01\%} - P_y|$$

$$\text{Alternative: } PVBP = D_{mod} \times P \times 0.0001$$

Dollar price change for a 1 bp yield move

DERIVATIVES

Forward contract price

$$F_0 = S_0(1 + r)^T$$

$$\text{With continuous dividends: } F_0 = S_0 e^{(r-q)T}$$

S₀ = spot, r = risk-free rate, T = time, q = dividend yield

Put-call parity

$$C + \frac{X}{(1+r)^T} = P + S_0$$

C = call price, P = put price, S₀ = spot price, X = exercise price

r = risk-free rate, T = time to expiration

Intrinsic value and time value

Call intrinsic: $\max(S - X, 0)$; Put intrinsic: $\max(X - S, 0)$

Time value = Option price - intrinsic. ATM/OTM intrinsic = 0; deep ITM time value → 0 near expiry.

Lower bound on European options (no dividends)

$$\text{Call: } c \geq \max(S_0 - X(1+r)^{-T}, 0)$$

$$\text{Put: } p \geq \max(X(1+r)^{-T} - S_0, 0)$$

Enforces no-arbitrage. Below these, the option is mispriced relative to the synthetic.

Option payoff at expiration

Long call: $\max(S_T - X, 0)$; Long put: $\max(X - S_T, 0)$

S_T = price at expiry, X = strike. Short positions are the negative of long. Subtract premium paid for profit.

Forward price with discrete income or cost

$$F_0(T) = (S_0 - PV(I) + PV(C))(1+r)^T$$

I = discrete income (dividends, coupons) over T; C = carrying cost (storage). PV at risk-free rate. Income reduces the forward; cost raises it.

ALTERNATIVE INVESTMENTS

Capitalization rate (Cap rate)

$$V = \frac{NOI}{r_{cap}}$$

NOI = net operating income (stabilized), r_{cap} = cap rate

Cap rate = NOI / Value (inverse: value = NOI / cap rate)

NAV per share

$$NAV = \frac{\text{Total assets} - \text{Total liabilities}}{\text{Shares outstanding}}$$

Used for mutual funds, ETFs, private equity fund valuation

Hedge fund fee structure (2-and-20)

Mgmt fee = $m \times AUM$ (e.g. 2%). Incentive fee = $p \times \max(0, \text{Profit above hurdle})$ (e.g. 20%).

Net investor return = gross - both fees.

Debt Service Coverage Ratio (DSCR)

$$DSCR = \frac{NOI}{\text{Debt service}}$$

Debt service = annual principal + interest. DSCR > 1 means cash flow covers debt; CRE lenders typically require ≥ 1.20 – 1.30 .

Loan-to-Value (LTV)

$$LTV = \frac{\text{Loan amount}}{\text{Property value}}$$

Higher LTV = more leverage and credit risk. Typical max $\approx 80\%$ commercial; 95%+ residential with mortgage insurance.

Net Operating Income (NOI)

$$NOI = \text{Effective Gross Income} - \text{Operating Expenses}$$

Excludes financing (interest), income tax, depreciation, and amortization. Foundation of cap-rate valuation.

Information ratio

$$IR = \frac{R_p - R_B}{\sigma_{R_p - R_B}} = \frac{\alpha}{\text{Tracking error}}$$

R_B = benchmark return, α = active return, TE = active risk

Jensen's alpha

$$\alpha_p = R_p - [R_f + \beta_p(R_m - R_f)]$$

Actual return minus CAPM-expected return

$\alpha > 0$ means manager added value beyond compensation for risk

Two-asset portfolio variance

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$$

w = weights, σ = std devs, ρ_{12} = correlation coefficient

Capital Market Line (CML)

$$E(R_p) = R_f + \frac{E(R_m) - R_f}{\sigma_m} \cdot \sigma_p$$

Sharpe ratio of market is slope; uses total risk σ_p (not beta)

Treynor ratio

$$T_p = \frac{R_p - R_f}{\beta_p}$$

Excess return per unit of systematic risk (beta)

Compare with Sharpe (uses total risk σ_p)

CAPM / Security Market Line (SML)

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2}$$

Uses systematic risk only; SML plots expected return vs beta