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FORMULA SHEET

# CAIA LEVEL II

Chartered Alt. Investment Analyst

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FORMULAS

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TOPICS

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INSTITUTIONAL ASSET OWNERS

2 items

**Kelly criterion (optimal fraction to bet)**

Binary:  $f^* = \frac{p(b+1) - 1}{b}$ . Continuous:  $f^* = \frac{\mu - r_f}{\sigma^2}$ . Maximizes log-wealth; often run at  $f^*/2$  or  $f^*/4$  for drawdown control.

**Risk parity portfolio weights**

Inverse-vol approx:  $w_i \propto \frac{1/\sigma_i}{\sum_j 1/\sigma_j}$ . Equal RISK per asset (not equal capital). Full form uses MRC:  $MRC_i = w_i(\Sigma w)_i/\sigma_p$ . Typically levered 2-3x.

ASSET ALLOCATION

2 items

**Brinson active-return decomposition (allocation + selection)**

Active return = Allocation + Selection + Interaction.

Allocation:  $\sum_i (w_{p,i} - w_{b,i})R_{b,i}$

Selection:  $\sum_i w_{b,i}(R_{p,i} - R_{b,i})$

Isolates sector tilts vs security picks.

**Surplus and surplus return (asset-liability framework)**

$$S_t = A_t - L_t$$

$$R_S = \frac{R_A \cdot A - R_L \cdot L}{A - L}$$

Surplus = assets minus liabilities; surplus risk = vol of  $R_S$ . Pension and insurer programs optimize on surplus Sharpe, not pure-asset Sharpe.

RISK AND RISK MANAGEMENT

4 items

**Tracking error**

$$TE = \sigma(R_p - R_b) = \sqrt{\text{Var}(R_p - R_b)}$$

Stdev of active return. Sets the active-risk budget for benchmark-relative managers. Information ratio = active return / TE.

**Kaplan-Schoar PME (KS-PME)**

KS-PME =  $\frac{\sum_t D_t/M_t}{\sum_t C_t/M_t}$ .  $D_t$  = distributions,  $C_t$  = contributions,  $M_t$  = public index. >1.0 means PE beat benchmark; <1.0 means underperformed.

**Parametric VaR (normal)**

$$\text{VaR}_\alpha = -(\mu + \sigma \cdot z_\alpha)$$

$z_{0.95} = -1.645$ ,  $z_{0.99} = -2.326$ . Reported as positive loss. Underestimates HF / PE tails; use Modified VaR or CVaR for fat tails.

**Expected loss (credit risk)**

$$EL = PD \times LGD \times EAD$$

PD = probability of default. LGD = loss given default = 1 - recovery. EAD = exposure at default. Basel pillar 1; private-credit underwriting.

METHODS AND MODELS

4 items

**Hedge fund fund-of-funds effective fee**

Fees compound: FoF 1-and-10 on top of HF 2-and-20  $\Rightarrow$  ~3% mgmt + ~28% of gross alpha. Creates a hurdle the FoF must beat net-of-fees; drove post-2008 FoF decline.

**Convertible arbitrage delta hedge**

Hedge ratio = convertible delta  $\times$  conversion ratio. Short  $\Delta$  shares per bond; rebalance as delta moves (gamma). P/L from gamma, carry, spread tightening, rising IV.

**Merger arbitrage annualized return**

$R_{arb} = \frac{P_{offer} - P_{current}}{P_{current}} \times \frac{365}{\text{Days to close}}$ . Conditional on close; multiply by P(completion) for risk-adjusted.

**Modified Internal Rate of Return (MIRR)**

$$\text{MIRR} = \left( \frac{FV(+CF \text{ at } r_r)}{|PV(-CF \text{ at } r_f)|} \right)^{1/T} - 1$$

Reinvests positives at  $r_r$ , discounts negatives at  $r_f$ . Avoids multiple-IRR issues from non-conventional flows.

ACCESSING ALTERNATIVE INVESTMENTS

2 items

**Trend-following signal (simple moving average crossover)**

Long when  $MA_{short} > MA_{long}$ ; short when reversed. Position size =  $\frac{\text{Target vol}}{\sigma_t} \times \text{Signal}$ . Vol targeting keeps portfolio vol constant.

**Commodity futures total return decomposition**

$R_{futures} = R_{spot} + R_{roll} + R_{collateral}$ ;  $R_{roll} = \frac{F_{near} - F_{next}}{F_{near}}$ . Positive in backwardation, negative in contango.

DUE DILIGENCE AND SELECTING MANAGERS

1 item

**Information ratio and Fundamental Law**

$$IR = \frac{\alpha}{\omega} = \frac{\text{Active return}}{\text{Tracking error}}$$

Fundamental Law:  $IR \approx IC \sqrt{\text{Breadth}}$ . IC = skill correlation, breadth = independent bets per year.

**Conditional Value-at-Risk (CVaR / Expected Shortfall)**

$\text{CVaR}_\alpha = E[L \mid L \geq \text{VaR}_\alpha]$ ; for normal returns:  $\mu + \sigma \cdot \frac{\phi(z_\alpha)}{1 - \alpha}$ . Coherent (sub-additive); used in Basel III.

**Omega ratio**

$\Omega(r) = \frac{\int_r^\infty (1 - F(x)) dx}{\int_{-\infty}^r F(x) dx}$ . Prob-weighted gains above threshold  $r$  over losses below. Captures all moments; preferred for skewed/fat-tailed returns.

**Long-short market-neutral PnL**

$$R_{\text{port}} = w_L R_L - w_S R_S - c_{\text{borrow}} w_S - c_{\text{fin}} w_L$$

Dollar-neutral:  $w_L = w_S$ . Beta-neutral:  $\beta_L w_L = \beta_S w_S$ . Alpha is cross-sectional; PnL hinges on borrow cost.